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Spin-Axis Attitude Estimation by a Controlled Correlation Method

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One considers the most common optical attitude sensor system consisting of sun and infrared sensors used on spinning spacecraft in geostationary transfer orbit. Particularly the infrared sensors are subject to systematic bias errors that affect attitude estimation accuracy. A procedure that employs a very limited sample of measurements is presented, such that each bias error separately occurs only once and thereby can be considered as a random error on its own. This approach is applied to the inertial and body spin-axis estimation separately for two meteorological satellites in geostationary and intermediate transfer orbits. Attitudes obtained from the orbit determination assessment of the direction of larger velocity increments imparted to these satellites by means of propulsion engines of the satellites themselves are used to check the accuracy performance of the inertial attitudes estimated by means of the new method. The validity of the estimation of the mean spin-axis orientation in the body reference system is verified by looking at the resulting accuracy improvement of the inertial attitude relying on the corrected body spin-axis direction. In the two missions considered, high inertial attitude precisions of the order of 0.1 deg or better were obtained in this way.

Nomenclature

$\boldsymbol{D}$ = boresight direction of an infrared pencil beam sensor
$D*$ = zero-bias random error of $*$
$d*$ = differential of $*$
$\boldsymbol{E}$ = nadir direction vector
$E(*)$ = mathematical expectation of $*$
i = inclination of the skew slit of a V-slit sensor, rad
M = number of infra-red sensors
$\boldsymbol{M}$ = vector of zero-bias random errors
$\boldsymbol{m}$ = vector whose components are the μ_i angles, rad
$\boldsymbol{N}$ = spin-axis direction vector
$\boldsymbol{Q}_*$ = variance-covariance matrix of the zero-bias random vector $*$
$\boldsymbol{S}$ = sun direction vector
t_* = sensor pulse time epoch for crossing $*$
α = dihedral angle, rad
β = right ascension of a pencil beam sensor boresight, rad
γ = angle between sun and nadir directions, rad
δ = spin-axis declination, rad
ϵ_* = spin-axis tilt angle around the $*$ axis, rad
ζ = colatitude of the intersection of the skew and meridian slit of a sun sensor, rad
θ_e = Earth colatitude angle, rad
θ_s = sun colatitude angle, rad
κ = half an Earth chord angle measured by a pencil beam sensor, rad
λ = right ascension of the spin axis, rad
μ_* = declination of the boresight of pencil beam $*$, rad
$\bar{\mu}_*$ = isolated determination of μ_*, rad
$\tilde{\mu}_*$ = optimized estimation of μ_*, rad
ν_e = azimuth correction of the Earth center crossing by a tilted meridian sun slit, rad
ν_0 = azimuth correction of the sun crossing by a tilted meridian sun slit, rad
ρ = angular radius of the apparent Earth disc, rad
σ_*^2 = variance of the zero-bias random variable $*$
ϕ = solar azimuth measured by a V-slit sensor, rad
ω = spin rate, rad/s
$\hat{*}$ = actual angular measurement of $*$ as opposed to predictions based on a known attitude, rad

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Introduction

A FEW years ago EUMETSAT, the European organization for the exploitation of Meteorological Satellites, decided to procure for a second generation of spinning, geostationary, meteorological satellites: Meteosat Second Generation (MSG). The first launch (MSG1) took place on 28 August 2002. The last launch in the series of probably four satellites or more is scheduled to occur well beyond 2010. The transfer from an elliptical transfer orbit into the geostationary orbit via a few intermediate orbits is accomplished by firing two bipropellant main engines. Thrust directions are parallel to the satellite structural symmetry axis, which in turn is equal to the body direction of the intended spin axis. The accuracy of the orientation of the velocity increments, which is in fact equal to the mean inertial spin-axis orientation during a maneuver, is rewarded in mission lifetime. For example, an attitude error of 1 deg on the 1500 m/s to be imparted to the satellite is equal to half a year of inclination control. The motivation to achieve best accuracies employing the sensor system considered also applies to a large family of other geostationary satellite missions, which are at least spin stabilized in the transfer orbit. The sensor combination was also employed by the space probe GIOTTO, which flew to Halley's comet, and on CONTOUR1, which aimed for Encke's comet. In the latter two cases the sensor system was operated in the geocentric orbit to enable accurate orientation of the perigee kick motor.

The infrared (IR) telescope or pencil beam sensors generate time pulses at the time of the transition space to Earth (s/e) and Earth to space (e/s), but these pulses are systematically offset in time. This offset is a function of the specific sensor electronics, of the seasonal IR radiance profile of the Earth atmosphere, and also of the sensor scan path through the apparent Earth, which itself is determined by both the attitude itself and the satellite orbital position. The sensing principle is explained by Wertz (Ref. 1, pp. 244–249). This explanation is also given by Baldasini-Fontana et al.,[2] who describe the origin of systematic errors, the IR radiance profiles, and their application too. The size of the time offsets translate into rotation angles varying between 0.4 and 1.5 deg, which on their own are functions of the spin rate. These delays can partly be calibrated, but the attempt to determine the residual bias (often as large as + or −0.2 deg) by means of extended Kalman filters in two studies, Huynh et al.[3] and

Huynh et al.,[4] was not conclusive. Even if the filters converged, the attitudes obtained were always worse than those obtained without bias determination. It even appeared that simple approaches like those published later by Shuster,[5] which ignore the biases, gave equally good or better results than the extended Kalman filter with bias estimation. Further similar studies and the practical guidelines given by Wertz (Ref. 1, Chap. 14) gave no new insights. Therefore, it was decided by the European Space Operations Centre (ESOC) to implement a trial-and-error procedure, which consisted of comparing the flatness of error residual plots of the different IR sensors pulses for a larger number of attitudes. The attitude for which all of the residuals were flattest was taken as estimate. Although this procedure was manpower intensive and time consuming, it was a breakthrough. Attitude determination accuracies improved in general by a factor of five. Unfortunately, a description of this method and its results has been sealed in internal flight dynamics reports and has not been published. Therefore, it will be taken up in more detail in the next section. In all that follows this estimation method will be named the residual analysis procedure.

The present paper presents an alternative to the residual analysis procedure, whose operational implementation is simpler and its applicability more general. This method is in fact a general estimator concept, which reduces as much as possible the effect of high error correlation between similar measurements of the same measurement devices. This new method is called here the controlled correlation method. "Least correlated" is not taken in the strict mathematical sense because one will take at least and normally at most one smoothed measurement of each possible type and source (sensor), even if these measurements are correlated nevertheless. The controlled correlation is a batch algorithm in two separate steps. In the first step one determines the best selection of measurement epochs to achieve highest accuracy. In the second step a small set of single pseudomeasurements generated by smoothing physical data at the best epochs are input in an optimal estimation.

One will first describe the basic geometrical context of the spin-axis attitude in inertial space and develop the measurement equations for a conventional optical sensor system consisting of sun and Earth IR pencil beam sensors. Thereafter, the best measurement selection and optimal estimation will be detailed. In the last section, results of the controlled correlation method are compared with those of the residual analysis procedure using operational data. Second, one will derive the equations for the determination of the direction of the largest principal moment of inertia in the spacecraft body reference system. This information was essential on MSG1 to achieve high accuracies because the knowledge of the mass properties in between orbital maneuvers was poor. Contrary to inertial attitude determination, which is applicable in general to all of the missions enumerated before, the determination of the body direction of the largest principal moment of inertia is a more specific problem, which is relevant if a succession of accurately oriented velocity increment maneuvers needs to be implemented. In that case one will be able to make use of the sensor measurements together with the inertial attitude knowledge acquired by evaluating the performance of a previous nonnegligible velocity increment maneuver. Also for this, the controlled correlation method has been applied, and the variation of the inertial attitude accuracy caused by the inclusion of the computed spin-axis offset into the attitude determination has been determined.

Historical Perspective

Between 1970 and 1977 ESRO and its successor organization, the ESA, contracted out five studies for spin-axis attitude determination all involving different approaches of Kalman or Kalman–Bucy filtering applied to IR pencil beam or V-slit albedo data. These studies tried to exploit nutation dynamics and the changing geometrical configuration of sun, Earth, and spin-axis directions and had the purpose to overcome the drawbacks of constant bias errors. The results of the studies are reviewed by Fraiture.[6] It was concluded that even in the cases where observability could definitely be verified, bias estimation did not improve the accuracy of the attitude itself, which suggested that there existed a statistical background for the fact that the attitude estimation accuracy went down by including more parameters in the estimation schemes. The theoretical reason was partly uncovered by the information dilution theorem formulated by Fraiture.[7] The information dilution theorem is in fact an uncertainty theorem, which states that trying to push the accuracy limits of estimation by adding complexity to an estimation model in the presence of observability will increase the standard deviations of the estimates, but at the same time it will reduce the expected bias of the estimate; omitting systematic errors in the truth model[8] will necessarily lead to a biased estimate. In other words, small biases can only be resolved if the measurements are accurate enough; otherwise, the estimates that one intends to improve by augmenting the state, will degrade. The relative quality of "small" and "accurate" is a matter of the actual magnitude of the supplementary parameters either to be included or not, the size of the variance of the measurement errors, and the numerical properties of the estimator. There is thus a preferred estimator as explained by Rapoport and Bar-Itzhack,[9] and, of course, if the more intricate truth model is known, observable and appears acceptable by the preferred estimator criterion, omitting the additional parameters, will lead to loss of precision. In the case at hand, information dilution is an issue, as will be made clear in the last section before the discussion of the results. One has thus to ignore additional biases to circumvent information dilution but nevertheless take into account that for a given coverage period of a particular pencil beam the mean curve of measurement residuals should be flat or at least symmetrical with respect to the crossing of the earth center as a function of true anomaly, depending on the sensor specific electronic properties.

Consequently, it was decided in 1981, after having had the experience of six geostationary transfer orbits, to implement the residual analysis procedure. One starts with collecting the data of a complete coverage period for each IR sensor separately producing plots and mean slopes of the measurement error residuals corresponding to three attitudes, close to where one believes the true solution to be, on what is called the sun cone. The axis of this cone is the sun direction, and its semi-apex angle is the measured sun angle. One further adds three nearby attitudes on a narrower cone and another three on a wider cone. Parabolic fittings and corresponding interpolations are then made to find the point where the slopes of the residuals are flattest. If there are two or more differently inclined pencil beams, the procedure is combined, and the result is a compromise based on human judgment. In this way one obtains 1) an assessment of the bias values affecting the pencil beam measurements; 2) a qualitative indication whether the sun sensing is accurate within specification (which was not always the case); and 3) a good attitude whose accuracy could until recently not be surpassed by other methods.

The procedure is specific for spin-axis attitudes derived from IR sensing with finite measurement coverage periods. It requires the data of uninterrupted and complete sensor coverage intervals, undisturbed by any kind of maneuver. Operations have thus to be tailored to the implementation and practical feasibility of the procedure. This technique is hardly applicable when these conditions are not satisfied.

Controlled correlation attitude estimation introduced here for the first time perfects the exploitation of geometrical configuration changes. It is not subject to the constraint that a set of complete coverage periods be available. It relies on the observation that changes in geometrical configuration lead to real-time attitude determinations whose expected accuracy is a function of true anomaly. The high but numerically unknown degree of correlation among the attitude sensor measurement errors makes it impossible, in practice, to implement either a recursive or a batch estimation that correctly weighs the contribution of measurements of less favorable geometrical configurations. A posteriori analysis of data has repeatedly shown that accuracy achieved by using a small batch of measurements taken in an interval with a good geometrical configuration worsens if the batch is augmented with data coming from an interval with a less favorable geometry. This worsening can be amplified by using filters to which system or plant noise is added to avoid the freezing of the estimates. If emphasis is put on best accuracy, one should thus look for the best selection of all types of measurements taken only once. In this way the maximum amount of trustworthy information is employed, and unknown biases can be considered to act as random errors in parallel to jitter. The latter can be reduced by deriving smoothed pseudomeasurements. In the controlled correlation procedure one assumes that the best pragmatic compromise for sampling information affected by biases is achieved by a data collection consisting of only one single measurement of one type of each single measurement device taken once among all measurements available from all devices. The different epochs of these separate single pseudomeasurements are subject to a selection resulting from the optimization of the expected error covariance of the estimate. The procedure is then as follows:

1) One determines an initial attitude of uncertain accuracy.

2) By minimizing the cost function, one determines the different optimal epochs where measurements have to be taken on the basis of the previous attitude.

3) One computes smoothed pseudomeasurements at the epochs just obtained.

4) This limited set of pseudomeasurements is introduced in an optimal batch attitude estimation normally requiring the application of differential correction.

Steps 1 and 2 can be performed in advance. The actual estimation step, which with modern computers lasts a fraction of a second, can be initiated as soon as the data at the last optimal measurement epoch have been collected.

In practice, precise attitude estimation cannot be performed without careful treatment of sensor alignments with respect to the true spin axis. This implies the merging of spin-axis alignment with structural and optical alignments. The total angular size of the spin-axis deviation from its target orientation in the body system is described by the wobble or coning angle. The exact orientation of the largest principal moment of inertia in the body coordinate system is defined by two small rotation angles, which are named spin-axis tilt angles. Prelaunch sensor mechanical and optical alignments are usually known with a quite reasonable accuracy. The effect of the launch shock on alignments, which is addressed in Shuster and Pitone[10] for three-axis-stabilized satellites, and which is among others because of the new structural and thermal equilibrium in space, can be assumed to have a magnitude that can be neglected for spinners. The best verified attitude accuracies for such satellites achieved to date and known to the author hardly exceed 0.05 deg, even when using star sensors. Not only information dilution but also observability preclude improved estimations of sensor alignments with conventional sun and IR sensing. This was confirmed in an analysis described by Gonzales.[11] In general, it is therefore important to implement the prelaunch detailed alignment information, resulting from careful and costly efforts at the manufacturers, provided that contractual and organizational measures have been taken in time to ensure a clean information transfer. This information transfer is especially important for the wobble and the corresponding spin-axis tilt angles because their sizes usually exceed by far the sensor alignment corrections in the body reference system. Results of dynamical balancing obtained shortly before launch are often not communicated to the flight dynamics team, because after balancing the system specification is contractually satisfied. But even then, it appears mandatory to include all information that is anyway practically and formally available into the attitude estimation if it augments the chance not to miss the contractual attitude estimation accuracy specification, especially if a major mission impact is at stake. And this is the case in geostationary transfer orbits. One should not forget that system requirement allowances for coning angles usually are in the range of 0.1–0.3 deg, which is really not negligible. If one aims at best accuracies, this is only acceptable for attitude determination if spin-axis tilt angles are taken into account, even if they are within specification. The spin axis tilt is further affected if there are large quantities of liquid propellants for which one is unable to produce exact prelaunch predictions of their distribution in the tanks as a function of mixing ratios and the actual operational maneuver sizing. This is the reason why the determination of the spin-axis orientation in the body system is important in flight, in order to correct the sensor alignments accordingly. The determination of the spin-axis misalignment will be discussed for MSG, which is equipped with a V-slit sun sensor as well as with three IR pencil beams.

Inertial Attitude

To avoid misunderstanding, it must be stressed that the estimation of the spin-axis orientation in this paper is for a nonnutating spacecraft, a condition that, under the influence of nutation dampers, is usually achieved very quickly after a maneuver. This means that the designation *spin axis* always refers to the direction of the angular momentum which coincides with the direction of the largest principal moment of inertia in the body system.

Measurement Equations

Let $\boldsymbol{S}$ be a unit vector from the spacecraft to the sun, let $\boldsymbol{E}$ be a unit vector from the spacecraft to the Earth center, and let $\boldsymbol{N}$ be the unknown spin axis as shown in Fig. 1. These vectors are subject to the following scalar products:

$$\boldsymbol{N} \cdot \boldsymbol{S} = \cos\theta_s \tag{1}$$

$$\boldsymbol{N} \cdot \boldsymbol{E} = \cos\theta_e \tag{2}$$

The sun angle can be considered to be measured directly. The angle θ_e is also known as the nadir angle. This angle is measured indirectly if IR pencil beams or albedo sensors are used. Sun and Earth angles are in the interval $(0, \pi)$, and therefore the sine of these angles is always positive or $+\sqrt{(1 - \cos^2\theta)}$. Almost all sensing systems allow the measurement of the angle α rotated by a reference meridian from the intersection with the sun direction up to the intersection with the Earth center. The availability of the dihedral angle α is assumed here. It is geometrically defined between 0 and 2π and is subject to the cosine rule:

$$\boldsymbol{S} \cdot \boldsymbol{E} = \cos\gamma = \cos\theta_s \cos\theta_e + \sin\theta_s \sin\theta_e \cos\alpha \tag{3}$$

as well as to

$$\boldsymbol{N} \cdot (\boldsymbol{S} \times \boldsymbol{E}) = \sin\theta_s \sin\theta_e \sin\alpha \tag{4}$$

The ith IR Earth sensor provides the angle $2\kappa_i$, which corresponds to the angle swept out by the spacecraft when moving the pencil-beam line of sight from the initial intersection with the apparent Earth disc (s/e) to the point where the beam leaves the apparent disc (e/s). The mean Earth radius up to the mean IR horizon is approximately 6402 km, as can be derived from data given in the relevant NASA report.[12] The geometry of a pencil beam scanning the Earth is represented pictorially in Fig. 2. If the ith pencil beam has an elevation μ_i with respect to the spacecraft equator, the measurement κ_i is subject to the following equation:

$$\cos\rho = \sin\mu_i \cos\theta_e + \cos\mu_i \sin\theta_e \cos\kappa_i \tag{5}$$

The angle κ_i is always smaller than $\pi/2$.

The equations just introduced give the full and well-known picture (for instance, see Shuster[5] or Bird et al.[13]) of the overall system to be considered. The linearity in the components of $\boldsymbol{N}$ and the linear

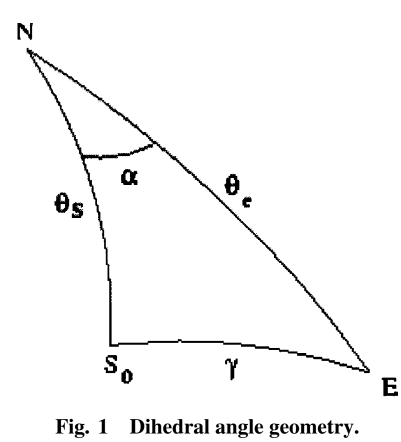

Fig. 1 Dihedral angle geometry.

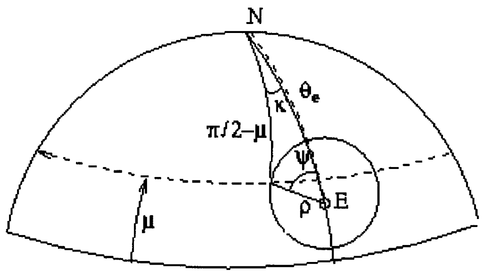

Fig. 2 Pencil-beam sensing geometry of the Earth horizon crossing.

independence of Eqs. (1), (2), and (4) allow the computation of the attitude with very high robustness. One calls *geometric attitude* estimates the attitudes that are obtained by solving this linear system directly. The geometric attitude calculation is based on a consistent value for θ_e derived from the application of Eqs. (3) and (5). If there are M pencil beams providing simultaneous measurements, one has $M+1$ equations to compute θ_e optimally. Such an optimal θ_e estimation has been in use in the geometric estimations of ESOC for more than 25 years, and a description of the process can be found in Bird et al.[13] The geometric estimation is able to provide reliable initial attitudes for advanced nonlinear attitude determination methods.

The separate computation of the intermediate parameter θ_e subdivides the preceding system of equations in two parts in a way that is suboptimal from a numerical point of view. It further suggests that the measurements of α and κ occur at simultaneous epochs, that is, are considered in pairs with the same time label. This limitation, which constrains the optimization of the controlled correlation, is artificial.

To characterize the optimal treatment, one considers the preceding system of equations in the case of only one pencil-beam sensor. There are three unknowns in this system, namely, λ, δ for the unique definition of the spin axis:

$$N' = [\cos\lambda\cos\delta, \sin\lambda\cos\delta, \sin\delta]$$

where an accent denotes transposition, plus θ_e. This stands against five equations, but altogether only three measurements, namely, θ_s, α, and κ. Hence, if this equation system is linearly independent (at the level of Jacobians) two of the five equations must be independent constraint equations. But if this were so, the knowledge of one unknown would allow determination of the other two unknowns (with an enumerable number of ambiguities) by means of the two independent constraint equations. This is geometrically inconsistent with the two degrees of freedom of an unconstrained spin-axis orientation. It is in fact Eq. (3) that has to be removed, if one wants to obtain an unambiguous result. This preserves the information contained in $\sin\alpha$ in Eq. (4), which is not dependent on the other measurements, whereas $\cos\alpha$ could in principle be computed from Eq. (3), using available values of κ and θ. If one wants to keep Eq. (2), which does not contain any measurement, one is plagued by unnecessary constraint equations as there will be a need for as many θ_e as one needs α and κ measurements at different times. For reasons of numerical simplicity and estimation optimality, the θ_e angles have to be eliminated.

One has to rearrange the system of equations to eliminate θ_e such that measurements are separated from the unknowns in order to satisfy the conditions required in the Gauss–Markov theorem (for instance, see Bard[14]). For one sun sensor and M Earth sensors providing each only one measurement of each type, one is left with

$$\boldsymbol{N}\cdot\boldsymbol{S}_0 = \cos\theta_s \tag{6}$$

$$\frac{\boldsymbol{N}\cdot(\boldsymbol{S}_{i1}\times\boldsymbol{E}_{i1})}{\left[1-(\boldsymbol{N}\cdot\boldsymbol{E}_{i1})^2\right]^{\frac{1}{2}}\left[1-(\boldsymbol{N}\cdot\boldsymbol{S}_{i1})^2\right]^{\frac{1}{2}}} = \sin\alpha_i \tag{7}$$

$$\frac{\cos\rho_{i2}-\sin\mu_i(\boldsymbol{N}\cdot\boldsymbol{E}_{i2})}{\cos\mu_i\left[1-(\boldsymbol{N}\cdot\boldsymbol{E}_{i2})^2\right]^{\frac{1}{2}}} = \cos\kappa_i \tag{8}$$

where the first subscript i refers to the pencil beam used, or $i = 1,\ldots,M$. A second subscript j is attached to $\boldsymbol{E}$, $\boldsymbol{S}$, and ρ to indicate that they either refer to a dihedral angle with $j=1$ or to the scanned Earth chord with $j=2$ all considered at the epoch t_{ij}. The sun angle measurement is taken at a time t_0. Needless to say, the measurements are subject to errors, or $\theta=\theta_0+D\theta$, $\kappa_i=\kappa_{i0}+D\kappa_i$ and $\alpha_i=\alpha_{i0}+D\alpha_i$. The subscript zero identifies the true angle. Not using explicit forms of the measurements in the right-hand side of Eqs. (6) to (8) leads to a negligible error, as can be verified by making a Taylor expansion up to the fourth order of a sine or cosine of one of the measured angles and takes the expectation of this approximation for the incurred bias error.

A useful observation about observability can be made here. When taking only one measurement of each kind, the measurement equation system consists of $2M+1$ equations. These equations have a formally different left-hand side. One can thus conceive an extension of these equations to take N unknown biases into account with the intention to estimate them. But one has already to solve for the two unknown attitude parameters, and hence it is necessary but not sufficient that

$$N+2 \le 2M+1$$

in order that the biases be observable at all. If one includes further measurements inside a pencil-beam coverage interval, one could at a first glance hope to augment this number slightly. But unfortunately the geometrical measurement quality of the Earth chord degrades rapidly towards the middle of coverage, which corresponds to an extremum for the chord value. This leads to a singularity, which is a further reason to limit oneself to the selection of only the best measurement taken only once.

Error Covariance of the Measurements

Even if one takes different types of measurements from different sensors only once, they are thereby not necessarily uncorrelated. Here correlation results from the linear combination of single uncorrelated physical measurements in order to get usable angles. All relevant measurements rely on the generation of time pulses, except maybe the sun angle, which can be measured, for instance, by a one-axis digital sun sensor. But also then a slit sensor perpendicular to the spacecraft equator (called meridian slit) is required to trigger the measurement readout of the digital sensor and is indispensable to initiate the rhumb angle timing for attitude reorientation maneuvers with thrusters. This slit is also necessary to be able to generate dihedral angle measurements applicable to the different epochs and corresponding nadir directions. Assume that the triggering error on the meridian slit is mainly a bias equal to Dt_m. If the sun angle results from a V-slit sensor (Ref. 1, pp. 178, 179, 218–221, and 716–718), the triggering time t_{0s} of the skew slit is subject to the bias error Dt_{0s}. With such a sensor the sun colatitude is normally derived from the solar azimuth angle $\phi=\omega(t_{0m}-t_{0s})$ shown in Fig. 3. Similarly the (s/e) pulse of the ith pencil beam is assumed to be in error by $Dt_{i,s/e}$, while the (e/s) pulse is subject to an error $Dt_{i,e/s}$. All of these errors are assumed to be correlated and to remain more or less constant over an IR sensor coverage interval excluding its beginning and end. At start and end of coverage, the diamond-shaped bolometer detector does not completely enter optically into the apparent Earth disk, giving rise to the pagoda effect

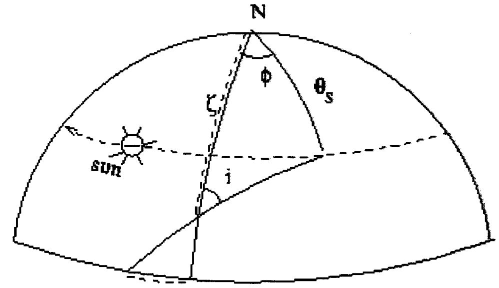

Fig. 3 V-slit sensing geometry.

described in the book edited by Wertz (Ref. 1, pp. 336–339). The elements of the vector $DM = [D\phi, D\alpha_1, D\kappa_1, \ldots]'$ of errors on the separate angular measurements are then defined by

$$D\phi = \omega(Dt_{0m} - Dt_{0s}), \qquad D\alpha_i = \omega[0.5(Dt_{i,s/e} + Dt_{i,e/s}) - Dt_{0m}]$$

$$2D\kappa_i = \omega(Dt_{i,s/e} - Dt_{i,e/s})$$

The corresponding variances are $\sigma_m^2 = E(Dt_{0m}^2)$, $\sigma_s^2 = E(Dt_{0s}^2)$, $\sigma_{i,s/e}^2 = E(Dt_{i,s/e}^2)$ and $\sigma_{i,e/s}^2 = E(Dt_{i,e/s}^2)$. These are variances of biases considered to be statistical independent random errors as they could occur, if one made a very large series of similar satellites under similar conditions with attitude sensors manufactured and mounted in the same way, etc., and where these biases have the expected value zero. The weakness of this approach is that these variances cannot be measured empirically. They either result from error budgets, or from the post factum evaluation of comparable missions.

The error covariance matrix Q_M in the simplified case of a V-slit sun sensor and one Earth sensor is equal to

$$Q_M = E(DM\,DM')$$

$$= \omega^2 \begin{pmatrix} \sigma_m^2 + \sigma_s^2 & -\sigma_m^2 & 0 \\ -\sigma_m^2 & 0.25\left(\sigma_{i,s/e}^2 + \sigma_{i,e/s}^2\right) + \sigma_m^2 & 0.25\left(\sigma_{i,s/e}^2 - \sigma_{i,e/s}^2\right) \\ 0 & 0.25\left(\sigma_{i,s/e}^2 - \sigma_{i,e/s}^2\right) & 0.25\left(\sigma_{i,s/e}^2 + \sigma_{i,e/s}^2\right) \end{pmatrix}$$

which is obviously not free of correlation.

Error Covariance of the Estimate

To adapt the system of observational equations to a matrix formulation adequate for error covariance analysis, one represents the $2M + 1$ equations corresponding to Eqs. (6–8) as the vector equation

$$F = Y \tag{9}$$

with the components

$$f_1 = N \cdot S_0, \qquad y_1 = \cos\theta_s$$

$$f_{2i} = \sin\alpha_i = y_{2i}, \qquad f_{2i+1} = \cos\kappa_i = y_{2i+1}$$

The $(2M + 1) \times (2M + 1)$ error covariance matrix Q of the measurements can be derived as in the preceding subsection assuming that $g(\theta_s, \phi) = 0$ with $[(\partial g/\partial\theta_s)/(\partial g/\partial\phi)]\,\mathrm{d}\theta_s = u\,\mathrm{d}\theta_s = \mathrm{d}\phi$ and noting that $DM' = [u\sin\theta D\theta, \cos\alpha_1 D\alpha_1, \sin\kappa_1 D\kappa_1, \ldots, \sin\kappa_M D\kappa_M]$. One further defines the $(2M + 1) \times 2$ Jacobian matrix J as follows:

$$J = \begin{pmatrix} \partial f_1/\partial\lambda & \partial f_1/\partial\delta \\ \vdots & \vdots \\ \partial f_{2M+1}/\partial\lambda & \partial f_{2M+1}/\partial\delta \end{pmatrix}$$

If the subscript 0 denotes the use of a known attitude and corresponding error free parameters, then for a Gaussian distribution of errors the Fisher information matrix is $J_0' Q^{-1} J_0$, and the corresponding Cramer–Rao bound is its inverse. The latter is in fact the covariance matrix C_{N_0} of the minimum variance estimates of the corresponding unknowns gained from an equation system satisfying the Gauss–Markov theorem for any unbiased error distribution in the case that N is equal to N_0. This means that

$$C_{N_0} = \left(J_0' Q^{-1} J_0\right)^{-1} = \begin{pmatrix} \sigma_\lambda^2 & \sigma_{\delta,\lambda} \\ \sigma_{\delta,\lambda} & \sigma_\delta^2 \end{pmatrix}$$

The expected attitude accuracy or cost function value for a given selection of measurement times $t_{11}, t_{12}, \ldots, t_{M1}, t_{M2}$ is thus simply

$$\Delta = \left[\sigma_\delta^2 + (\cos\delta)^2\sigma_\lambda^2\right]^{\frac{1}{2}} \tag{10}$$

Optimal Measurement Epochs

Different methods for finding the vector of optimal measurement epochs

$$T' = [t_{11}, t_{12}, t_{21}, t_{22}, t_{31}, t_{32}]$$

which minimizes the expected attitude accuracy $\Delta(T)$ have been tried out and applied to the recorded live data of the predecessor European satellite of MSG1, named Meteosat Transitional Program (MTP) as well as to simulated data anticipating the MSG1 case. The results were independent of the approach used and clearly indicated that the optimal measurement epochs in this particular case were always located either at the start or at the end of the usable IR sensor coverage. The usable IR coverage is the time interval where the measurements of a pencil-beam sensor are not subject to the pagoda effect mentioned earlier. As a rule of thumb, one discards measurements where the opening angle Ψ defined in Fig. 2 is smaller than 45 deg in geostationary transfer orbits. A heuristic explanation for the position of the optimal measurement epochs can easily be given. If one computes the differential of Eq. (4) with respect to θ_e and κ, one finds upon elimination of $\cos\kappa$ in the denominator

$$\mathrm{d}\theta_e = -\frac{\cos\mu\sin^2\theta_e\sin\kappa}{\sin\mu - \cos\rho\cos\theta_e}\,\mathrm{d}\kappa \tag{11}$$

But if the pencil beam intersects the IR horizon such that the arc ρ is at a right angle to the meridian containing the apparent Earth center, then $\sin\mu = \cos\rho\cos\theta_e$, which means that a negligible error $\mathrm{d}\kappa$ leads to $\mathrm{d}\theta_e = \infty$. This singularity does not occur at the point where the boresight of the IR sensor crosses the Earth center (except if $\mu = 0$), contrary to the impression a confusion between small and great circle arcs can lead to. In the particular case of MSG1, it therefore happens that coverage start or end are the more optimal measurement locations. Earth chords are not equally accurate on both sides of a coverage interval. The optimal choice has still to be sorted out in a practical case. For low Earth orbits and other attitudes with very long coverage periods, this will not necessarily hold true. Specific and systematic analysis for MTP showed that the optimal epochs for the dihedral angles are equal to the optimal epochs for the Earth chords for each pencil beam separately. This observation dramatically simplifies the controlled correlation procedure for all MSG because the selection of the best measurement epochs only involves eight combinations.

At this point it might be useful to compare this approach with the covariance analysis of Van der Ha.[15] The difference between the present approach and the one in the reference is fundamental because in Van der Ha[15] one considers simultaneous measurements, whereas here one has just stressed the optimal selection of mutually different measurement epochs. Another less important difference is the fact that θ_e has not been eliminated in Van der Ha[15] but is subject to an optimized choice similar to the one of Bird et al.[13] Considering today's improved tracking systems and resulting orbit precision, also orbit determination errors have now been neglected.

Determination of the Spin Axis in the Body System

Different attempts have been made to determine the tilt angles and other measurement biases together with the inertial attitude as reported by Fraiture.[6] In these studies simulation, estimation, and nutation dynamics are based on geometrical sensor pulse definitions. If L is the vector normal to the plane containing a sun sensor slit, then a geometrical definition of the pulse is obtained by requiring that L is rotated in such a direction that $S \cdot L = 0$. Similarly, if $D_i = [\cos\mu_i\cos\beta_i, \cos\mu_i\sin\beta_i, \sin\mu_i]'$ is the the boresight direction of the ith pencil-beam sensor, then either a (s/e) or (e/s) pulse occurs if $D_i \cdot E = \cos\rho$. As mentioned earlier, none of these studies led to satisfactory results, and this was corroborated in the report by Gonzales.[11] The situation changes fundamentally if one disposes of a reliable inertial attitude, which can be compared with the measurements.

To define the tilt angles, one has first to define a body system. The body system is introduced by the spacecraft manufacturer as reference for mounting distances and all alignments. In practice, the actual spin axis is already tilted in the manufacturer's body system. Starting from there, one defines a new body system in which the direction of the assumed direction of the largest principal moment of inertia is aligned with a coordinate axis, which is the z axis for MSG. The choice of the other axes is arbitrary, but locating the x axis in the direction of the intersection of the meridian sun sensor slit with the spacecraft equator seems to be the more adequate choice. In this reference system a further unknown spin-axis tilt can be represented by a succession of two rotations, the first being a rotation over the angle ϵ_y around the y axis denoted by $\boldsymbol{R}_y(\epsilon_y)$ and a second rotation over ϵ_x around the x axis denoted by $\boldsymbol{R}_x(\epsilon_x)$. If $\boldsymbol{u}_z$ is a unit vector along the z axis in the present body system, one checks that

$$\begin{aligned}\boldsymbol{R}_{\text{tilt}}\boldsymbol{u}_z &= \boldsymbol{R}_x(\epsilon_x)\boldsymbol{R}_y(\epsilon_y)\boldsymbol{u}_z \\ &= [-\sin\epsilon_y, -\sin\epsilon_x\cos\epsilon_y, \cos\epsilon_x\cos\epsilon_y]' \end{aligned} \tag{12}$$

The untilted axis does not play any kinematical role because the spin motion is a steady rotation ($\dot{\omega} = 0$) around the tilted axis, but, as said before, it belongs to the coordinate system in which sensor alignments are originally described.

If the spin-axis direction is known to a reasonable accuracy in inertial space, one can compute θ_s and θ_e based on the scalar products (1) and (2). In the practical problem at hand, the known spin-axis direction is subject to a total error of at most 0.1 deg 3σ in some unknown direction whose probability is assumed to be uniformly distributed between 0 and 2π around the given spin axis. Hence, the resulting variances for θ_s and θ_e are of the same order of magnitude, if errors on $\boldsymbol{E}$ and $\boldsymbol{S}$ can be neglected. The computation of $\sigma_{\theta e}$ and $\sigma_{\theta s}$ is addressed in the Appendix. Also the actual dihedral angles $0 \le \alpha_k < 2\pi$ can be computed by using Eqs. (3) and (4). All of these predicted values will differ from the physical measurements if there is a spin-axis tilt. Therefore, from now onwards, parameters representing physical measurements will be identified by a caret. One disposes of the measured values $\hat{\kappa}_k$, $\hat{\phi}$, and $\hat{\alpha}_k$. If one exploits all that is known, it will be shown that spin-axis tilt can be determined directly without having recourse to advanced estimation techniques, especially for MSG, where all IR sensor beams and the sun meridian slit are located on the same great circle.

Determination of ϵ_y

For all pencil beams grouped, a common systematic change in the pencil-beam inclination is equal to ϵ_y. Independent from this remark, one observes that the new value of μ_k gained from Eq. (5) in the tilted body system can be considered as the only unknown in an equation where all other parameters are available if the inertial attitude is known. Thereby ρ is assumed to be exact, but $\hat{\kappa}_k$ and θ_e are subject to errors to be taken into account. The tilted beam inclinations found this way do not say anything about the right ascension of the beam direction in the tilted system. Nevertheless, the known angles between the beams are not affected by the spin axis tilt, and thus $\boldsymbol{D}_i \cdot \boldsymbol{D}_j$ is known. Hence, the values of μ_i and μ_j permit the computation of the difference $\beta_i - \beta_j$ apart from the sign because

$$\boldsymbol{D}_i \cdot \boldsymbol{D}_j = \sin\mu_i \sin\mu_j + \cos\mu_i \cos\mu_j \cos(\beta_i - \beta_j) \tag{13}$$

If the pencil beams are originally at the same right ascension and the tilt is small, the expression $\beta_i - \beta_j$ corresponds to a small angle. This is the case for MSG. Within the small angle approximation one derives from Eq. (13) that

$$\mu_i - \mu_j = \mu_{i0} - \mu_{j0}$$

where the subscript zero is used to represent values of parameters and vectors before spin-axis tilt occurred. These inclinations are only known up to the alignment measurement tolerance permitted before launch and the inaccuracy of the optical axis of the sensor caused by the imperfection of the calibration of the offset of the center of sensitivity of the bolometer. If the standard deviation of these errors is not provided by the satellite manufacturer, one has to make a reasonable guess to obtain an equally reasonable weighting of the corresponding measurement equations. All of this leads to a very simple overdetermined system of $2M - 1$ equations in M unknowns, namely,

$$\begin{aligned} \mu_1 &= \bar{\mu}_1 \\ &\ldots \\ \mu_M &= \bar{\mu}_M \\ \mu_1 - \mu_2 &= \mu_{10} - \mu_{20} \\ &\ldots \\ \mu_{(M-1)} - \mu_M &= \mu_{(M-1)0} - \mu_{M0} \end{aligned}$$

or abbreviated in matrix form:

$$\boldsymbol{Am} = \boldsymbol{B} \tag{14}$$

where $\boldsymbol{A}$ has dimension $(2M - 1) \times M$ and $\boldsymbol{B}$ is the vector representing the known member with the observed stochastic variables. With $\bar{\mu}_i$ one denotes the direct separate solutions of Eq. (5). The stochastic nature of $\boldsymbol{B}$ originates from the $2M + 1$-dimensional vector of uncorrelated errors $\boldsymbol{DG}' = [D\hat{\kappa}_1, \ldots, D\hat{\kappa}_M, (\cos\delta)D\lambda, D\delta, D\mu_{10}, \ldots, D\mu_{M0}]$. This yields the diagonal covariance matrix $\boldsymbol{Q}_g = E(\boldsymbol{DGDG}')$. After having worked out $\boldsymbol{DB} = \boldsymbol{CDG}$, the computation of the correlated optimal tilted inclinations $\tilde{\mu}_i$ of the pencil beams grouped in the vector $\tilde{\boldsymbol{m}}$ and the corresponding covariance matrix $\boldsymbol{Q}_{\tilde{\mu}}$ is trivial.

Controlled correlation means that one uses M measurements κ_i just once from each sensor and this at the location where they are able to provide the best attitude. These locations can be found by determining the epoch vector $\boldsymbol{T}$ discussed earlier. The tilt angle ϵ_y is then simply the weighted mean of the change of the M newly determined pencil-beam inclinations. Therefore, if $\boldsymbol{F}'$ is equal to the vector [1, 1, 1], the optimal value of ϵ_y is found from the basic matrix equation in one unknown, namely, $\boldsymbol{F}\epsilon_y = \tilde{\boldsymbol{m}} - \boldsymbol{m}_0$ yielding the weighted mean

$$\epsilon_y = \left(\boldsymbol{F}'\boldsymbol{Q}_{\tilde{\mu}}^{-1}\boldsymbol{F}\right)^{-1}\boldsymbol{F}'\boldsymbol{Q}_{\tilde{\mu}}^{-1}(\tilde{\boldsymbol{m}} - \boldsymbol{m}_0) \tag{15}$$

where the hidden correlation between $\tilde{\boldsymbol{m}}$ and $\boldsymbol{m}_0$ has been neglected.

Determination of ϵ_x Using the Solar Azimuth

For the determination of ϵ_x, one will have to find the appropriate way to include sun sensor data. In this paper one assumes that θ_s is derived from the solar azimuth angle ϕ using the trigonometric equation

$$\sin\zeta\cot\theta_s = \cos\zeta\cos\phi + \cot i \sin\phi \tag{16}$$

which is easily derived from Fig. 3, assuming that the meridian slit is truly meridian. For MSG $\zeta = \pi/2$ applies nominally, and consequently one has derived in the preceding section that $\epsilon_y = \pi/2 - \zeta$. A further rotation by ϵ_x around the tilted intersection point of the slits is required to enable a full description of the the spin-axis tilt. This rotation means that the meridian slit becomes a skew slit canted itself by ϵ_x at the slit intersection point. This is shown in Fig. 4. It

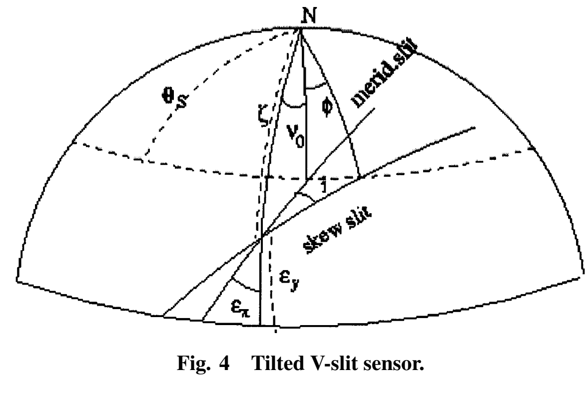

Fig. 4 Tilted V-slit sensor.

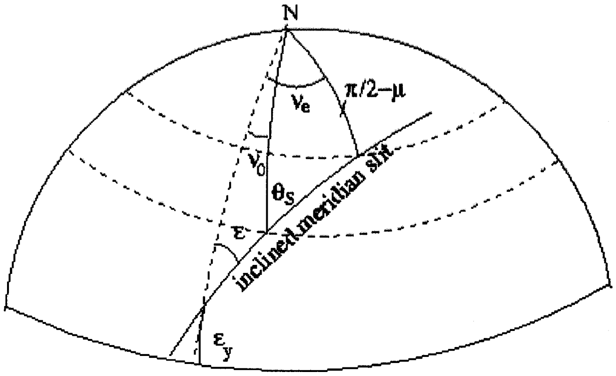

Fig. 5 Crossing delay angles at the inclined meridian slit.

implies that there is a small fictitious azimuth ν_0, which goes from a new conceptual meridian through the physical slit intersection to the point where the misaligned meridian slit intersects the sun direction. The angle ν_0 is computed by adapting Eq. (16), or

$$\cos\epsilon_y \cot\theta_s = \sin\epsilon_y \cos\nu_0 + \cot\epsilon_x \sin\nu_0 \tag{17}$$

The measured angle $\hat{\phi}$ is not equal to ϕ computed on the basis of the known attitude. According to Fig. 5, the actual solar azimuth for the skew slit becomes $\nu_0 + \hat{\phi}$ after tilting, and Eq. (16) can be rewritten as

$$\cos\epsilon_y \cot\theta_s = \sin\epsilon_y \cos(\nu_0 + \hat{\phi}) + \cot(i + \epsilon_x)\sin(\nu_0 + \hat{\phi}) \tag{18}$$

without approximation. The values of ϵ_y, θ_s, i, and $\hat{\phi}$ are corrupted by errors whose standard deviation must either be computed or have to be guessed on the basis of engineering error budgets.

For MSG both ϵ_y and ν_0 are small angles. Using a small angle approximation, Eqs. (17) and (18) combine to the simplified equation

$$\epsilon_x \approx \frac{\sin i \cot\theta_s - \cos i \sin\hat{\phi} - \epsilon_y \sin i \cos\hat{\phi}}{\cos i \cot\theta_s(\cos\hat{\phi} - 1) - \sin i \sin\hat{\phi}}$$

Unfortunately, the result of this approximation is too crude to be employed as estimate in a practical problem. But it is adequate to initiate a very basic differential correction in the two unknowns ν_0 and ϵ_x of the two equations (17) and (18).

Determination of ϵ_x Using the Dihedral Angles

One uses again the fact that on MSG all optical attitude sensors are located on the same great circle, which originally was a meridian. If the satellite is tilted, the occurrence of the meridian sun pulse will be anticipated by ν_0, as shown in Fig. 5. The crossing time of the IR sensor boresight through the sun meridian, and thereafter the horizon crossings, will all be delayed by the same rotation angle ν_e. The measured dihedral angle will thus be $\hat{\alpha}_{ej} = \alpha_{ej} - \nu_{ej} + \nu_{0j}$, where α_{ej} is derived from the known attitude. Looking at Fig. 5, one realizes that Eq. (16) can be applied to link ν_e to $\tilde{\mu}$. Applying a small angle approximation to the resulting equation yields

$$\nu_{ej} \approx \epsilon_x \tan\tilde{\mu}_j \tag{19}$$

Approximating Eq. (17) at the epoch where α_{ej} is measured and where θ_{sj} applies, one gets

$$\nu_{sj} \approx \epsilon_x \cot\theta_{sj} \tag{20}$$

taking the natural drift of the apparent sun into account. By abbreviating $(\hat{\alpha}_{ej} - \alpha_{ej})$ by $\Delta\alpha_j$ and $(\tan\tilde{\mu}_j - \cot\theta_{sj})$ by h_j, one checks that

$$\epsilon_x \approx \frac{\sum_{j=1}^{M} k_j \Delta\alpha_j}{\sum_{i=1}^{M} k_i h_i} \tag{21}$$

for any arbitrary constants k_j provided they are not all zero. Numerical tests with the MSG sensor alignments show that the approximation where only one k_i is different from zero is more precise than the combined application of Eqs. (17) and (18). Nevertheless, the application of this approach to smoothed live data gave rise to inconsistencies.

From a covariance analysis made by the author for the simultaneous analytical determination of ϵ_x and ϵ_y based on sensor pulse time combinations and a known but slightly inaccurate attitude, the expected variance of ϵ_x appeared to be up to 10 times larger than the one for ϵ_y. This is beyond the boundary where one would attempt to estimate ϵ_x at all because there is a nonnegligible probability that the potential estimation error is larger than the probable size of the true physical tilt angle ϵ_x itself (information dilution!). Nonetheless, one has succeeded to find a procedure that gives consistent results within a realistic magnitude range as described by de Juana-Gamo and Cegarra.[16]

One has employed the equations $\epsilon_x \approx (\Delta\alpha_j - \Delta\alpha_i)/(h_j - h_i)$ taking care that the smoothed measurements are close to the different optimal measurement epochs found before, but with the constraint that the values of the chords κ_j and κ_i are equal. In this way both pencil beams have an approximately equal scan path through comparable Earth latitudes. If the two pencil-beam sensors i and j have a similar response, one can hope to eliminate a larger part of the biases in α_j and α_i by choosing $k_j = 1$ and $k_i = -1$. In practice, one has only considered the measurements α_{e1}, α_{e2}, and α_{e3} in two algebraically, linearly independent measurements of the shape just described to obtain an estimate of ϵ_x. The solar azimuth angle has not been retained to estimate ϵ_x. Consequently, the controlled correlation procedure has only been applied partly in this special situation, where engineering insight has been given priority to decide on epochs and best weighting to apply to the measurement contributions.

Results

The inertial attitude determination by means of the controlled correlation method was initially tested by applying it to simulated data and further verified by using recorded telemetry data. Only results gained from operational data are summarized here. Before having had the experience of the MSG1 launch and early operations, the importance of the spin-axis tilt was not fully appreciated. The results reported in Table 1 reflect this state of affairs, for which it seems indeed sufficient to only take the manufacturer's tilt and detailed alignment data into account, because major liquid apogee engine firings (LAEF) are absent. The magnitude of the attitude estimation errors for the controlled correlation method and the residual analysis procedure show very similar values. The MTP data were revisited beyond the MSG1 launch, and the spin-axis tilt angles were determined for that satellite as well. Including the relevant tilt angles resulted in an attitude error of 0.04 deg for MTP at apogee 2 using the controlled correlation method, thus outperforming the residual analysis procedure slightly in all cases investigated (and not all reported here).

During the early orbit operations, beyond the first LAEF of MSG1, attitude estimates by all determination methods and procedures led to unsatisfactory attitude measurement residuals. Although one readily realized that the wobble angle differed from prelaunch predictions, analyses have been performed, and appropriate tools have been developed post factum. There is no direct way to verify that the tilt angles, which are determined, are actually

Table 1 Difference between orbit determination and sensor-based attitude estimation using manufacturer's alignments and tilt data

Attitude estimation method	MTP diff. at apogee 2, deg	MTP diff. at apogee 3, deg	MSG1 diff. at apogee 2, deg	MSG1 diff. at apogee 3, deg
Classical methods	0.203	0.126	0.32	0.15
Residual analysis	0.081	0.154	0.17	0.21
Contr. correl.	0.093	0.092	0.16	0.18

Table 2 Controlled correlation estimate errors before and beyond spin-axis tilt correction for MSG1

Orbit	Attitude error without tilt correction, deg	Attitude error with tilt correction, deg	Magnitude of wobble angle corrected for, deg
AP-3	0.18	0.09	0.17
AP-5	0.45	0.08	0.31
AP-9	0.55	0.05	0.46
AP-11	0.58	0.09	0.55

close to the truth. But the approximately correct tilt angles must lead to an attitude estimate improvement. And this occurred to an unexpected level. A summary of the result is displayed in Table 2. For the compilation of this table, one has collected the data of the last orbit before each of the four LAEF implemented with MSG1. The data are always sampled from all coverage intervals selected around apogee of orbit number n denoted by (AP-n). The reference attitude, which is assumed to be the formally correct attitude in the Tables 1 and 2, was determined each time after the the LEAF on the basis of the velocity increment direction obtained from accurate orbit determination. If this direction is in error by 0.1 deg 3σ, what is claimed by the orbit determination team, it is hard to decide after tilt correction which fraction of the attitude error is as a result of inaccuracy of the velocity increment assessment or of the spin axis tilt determination. Anyway, there is convincing evidence that the spin-axis tilt estimation procedure just explained is correct and worth implementing.

Conclusions

An estimation procedure was conceived to mitigate the effect of imponderable biases in problems, where, for a constant state, the measurements are subject to a natural variation as a function of time. The method has been applied to the optical sensor data of two spin-stabilized meteorological satellites in the early orbit phase, once for the determination of the inertial spin-axis orientation and once for the determination of the detailed spin-axis tilt. The accuracy of the results outperforms the best method used hitherto and known to the author. Moreover, the new approach gets rid of the drawbacks of the already existing accurate method, by suppressing human intervention completely after data collection. The controlled correlation procedure could also be considered for selective measurement updates of slowly varying states subjected or not to automatic control. One can think, for instance, of three-axis attitude updates based on global-positioning-system baseline angle measurements with the aim to reduce effects of biases and enhance the accuracy of the measurement-based state updates.

Appendix: Assumed Standard Deviations

The calculation of the standard deviation of the sun colatitude θ_s error or the nadir angle θ_e error computed back from a known attitude subject to a 0.1 deg 3σ error is explained hereafter. Taking the nadir direction $\boldsymbol{E} = |e_x, e_y, e_z|'$ and the corresponding Earth colatitude as example, one observes that the relevant first-order errors are subject to

$$D(\cos\theta_e) = (-e_x \sin\lambda + e_y \cos\lambda)(\cos\delta) D\lambda - e_z \sin\delta D\delta$$

or abbreviated:

$$D\theta_e = \frac{[U_e(\cos\delta) D\lambda + V_e D\delta]}{\sin\theta_e}$$

By assumption the expected error variances for perpendicular attitude directions with equal metric measure are uncorrelated and equal, or

$$\sigma_N = \sqrt{E[(\cos^2\delta) D\lambda^2]} = \sqrt{E(D\delta^2)} = 0.1\big/\left(3\sqrt{2}\right) \text{ deg}$$

and consequently

$$\sigma_{\theta e} = \frac{\sigma_N \sqrt{U_e^2 + V_e^2}}{\sin\theta_e}$$

holds. For the sun colatitude one equally writes $\sigma_{\theta s} = \sigma_N \sqrt{(U_s^2 + V_s^2)} / \sin\theta_s$.

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